MATH 119: Midterm 1

Name: Key

Directions:

- Show your thought process (commonly said as ”show your work”) when solving each problem for full credit.
- If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
		50

1. Short answer questions:

   (a) Suppose you write

   $$(x+y)^2z^2 = x^2 + y^2z^2$$

   What are the two errors you made?

① x and y are terms. Cannot use laws of exponents on terms.

② $(x+y)^2$ generates three terms $x^2 + 2xy + y^2$ so z^2 needs to be distributed to each term by the distributive law, not just y^2

   (b) True or false: We can simplify

   $$\frac{(x+1)(x-2)+(x-2)(x+3)}{x+1}$$

   by crossing out the $x+1$.

No because $(x+1)$ is a part of a term in the numerator not a factor in the context of the entire numerator.

   (c) If $f(x) = x^2 - x$, evaluate $f(-x+h)$ and expand.

do not forget

$$f(-x+h) = (-x+h)^2 - (-x+h)$$

$$= \boxed{x^2 - 2xh + h^2 + x - h}$$

   (d) Suppose $f(x) = \sin(x)$. Do

   $$g(x) = \sin(x+\pi) \qquad h(x) = \sin(2x+\pi) = \sin\left(2\left(x+\frac{\pi}{2}\right)\right)$$

   have the same horizontal shift? If not, what are both $g(x)$ and $h(x)$'s horizontal shift?

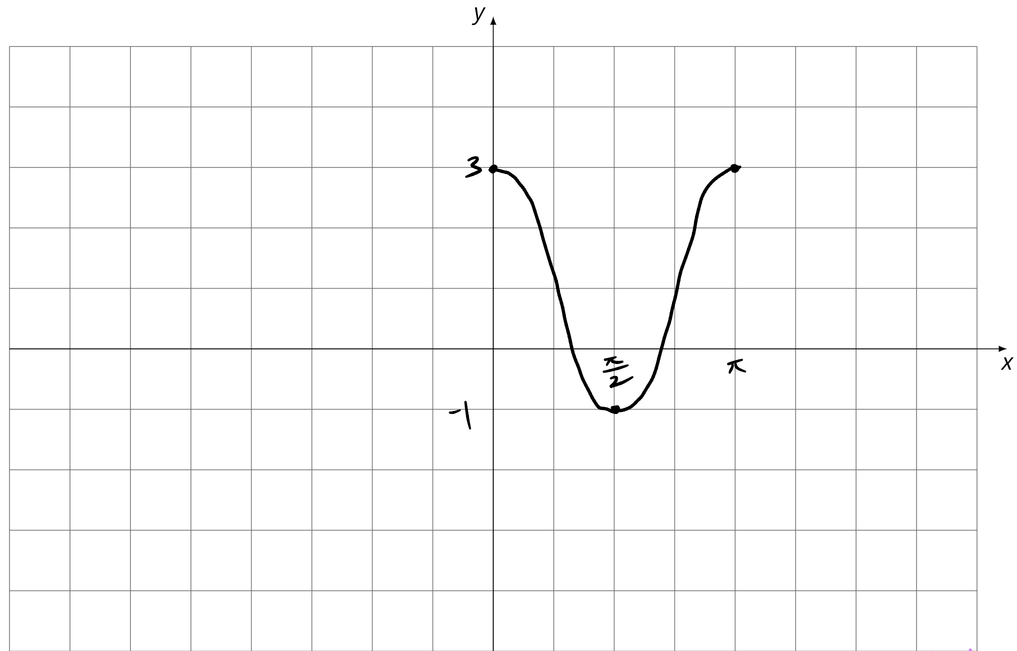
No. $g(x)$ is π units to left of $f(x)$.

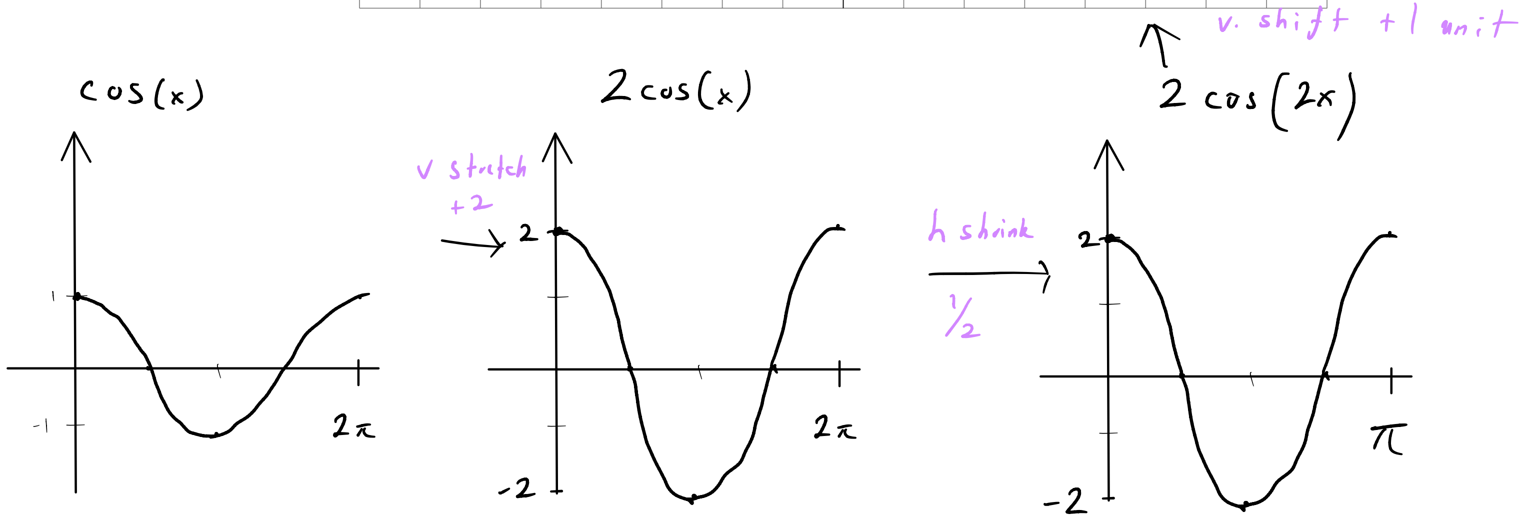
$h(x)$ is $\frac{\pi}{2}$ units to left of $f(x)$.

2. Suppose

$$f(x) = 1 + 2\cos(2x)$$

(a) Sketch a graph of $f(x)$.





(b) What is $f(\pi)$?

$$f(\pi) = 1 + 2\cos(2\pi) = 1 + 2 \cdot 1 = \boxed{3}$$

3. Follow the given instruction. Remember to use the relevant laws/properties.

(a) Simplify: $\left(\frac{x+1}{(x-1)}\right)^2 \cdot \left(\frac{(x-1)(x+1)}{x+2}\right)^{-2}$

\# 1 mistake: not stopping and looking for terms. You can't use LoE on terms. No Law out of the seven we know has any terms involved.

$$\overset{⑥}{=} \left(\frac{x+1}{x-1}\right)^2 \cdot \left(\frac{x+2}{(x-1)(x+1)}\right)^2$$

$$\overset{⑤}{=} \frac{(x+1)^2}{(x-1)^2} \cdot \frac{(x+2)^2}{(\underbrace{(x-1)(x+1)}_{\text{factors}})^2}$$

$$\overset{④}{=} \frac{(x+1)^2}{(x-1)^2} \cdot \frac{(x+2)^2}{(x-1)^2(x+1)^2} \overset{\text{frac}}{\underset{\text{law ①}}{=}} \frac{\cancel{(x+1)^2}(x+2)^2}{(x-1)^2(x-1)^2\cancel{(x+1)^2}} \overset{\text{LoE ①}}{\underset{\text{frac law ⑤}}{=}} \boxed{\frac{(x+2)^2}{(x-1)^4}}$$

(b) Expand: $(x-2)^2(x+3)+(x-3)(x+2)$

$(A-B)^2$

$$\overset{(A-B)^2}{\underset{\text{dist law}}{=}} (x^2-4x+4)(x+3) + (x-3)x + (x-3)2$$

$$\overset{\text{dist}}{\underset{\text{law}}{=}} (x^2-4x+4)x + (x^2-4x+4)3 + x^2-3x+2x-6$$

$$\overset{\text{dist}}{\underset{\text{law}}{=}} x^3-4x^2+4x+3x^2-12x+12+x^2-x-6$$

$$= \boxed{x^3-9x+6}$$

(c) Completely factor (you should have four factors): x^4-5x^2+4

Let $y=x^2$. Then

$$x^4-5x^2+4 \overset{\text{LoE } 3}{=} (x^2)^2-5x^2+4$$

$$= y^2-5y+4$$

$$\begin{matrix} 1 & -4 \\ 1 & -1 \end{matrix}$$

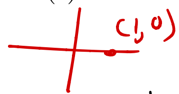
$$= (y-4)(y-1)$$

$$= (x^2-4)(x^2-1)$$

$$\overset{A^2-B^2}{=} \boxed{(x-2)(x+2)(x-1)(x+1)}$$

4. Let

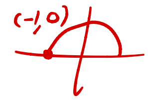
$$f(t) = \sin(t) \qquad g(t) = \cos(t)$$



Find the following:

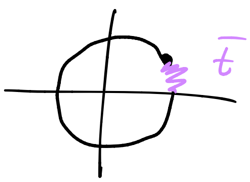
(a) $f(\pi \cdot g(0)) = f\left(\pi \cdot \cos(0)\right) = f(\pi \cdot 1)$

$= \sin(\pi)$



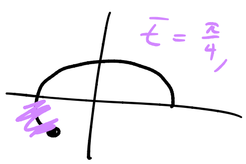
$= \boxed{0}$

(b) $f\left(\frac{-11\pi}{6}\right) = \sin\left(-\frac{11\pi}{6}\right) = +\sin\left(\frac{\pi}{6}\right) = \boxed{\frac{1}{2}}$



$\bar{t} = \frac{\pi}{6}$, sin + in I

(c) $g\left(\frac{5\pi}{4}\right) = \cos\left(\frac{5\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = \boxed{-\frac{\sqrt{2}}{2}}$



$\bar{t} = \frac{\pi}{4}$, cos − in III

(d) What is the domain of $f(t) + g(t)$?

$f(t) = \sin(t)$ has domain $\mathbb{R}$

$g(t) = \cos(t)$ has domain $\mathbb{R}$

$f(t) + g(t) = \sin(t) + \cos(t)$ has domain $\mathbb{R} \cap \mathbb{R} = \boxed{\mathbb{R}}$

(e) If $f(t) = -\frac{4}{5}$ and the terminal point of t is in Quadrant IV, what is $g(t)$?

Using $\sin^2(t) + \cos^2(t) = 1$

$\left(-\frac{4}{5}\right)^2 + \cos^2(t) = 1$

$\frac{16}{25} + \cos^2(t) = 1$

$\cos^2(t) = 1 - \frac{16}{25} = \frac{25}{25} - \frac{16}{25}$

$= \frac{25-16}{25}$

$\cos^2(t) = \frac{9}{25}$

$\cos(t) = \pm\sqrt{\frac{9}{25}} = \pm\frac{\sqrt{9}}{\sqrt{25}} = \pm\frac{3}{5}$

cos(t) is + in IV

so $\boxed{\cos(t) = \frac{3}{5}}$

goal: $x = \dots$

5. Solve for x and remember to check your work if necessary:

Get x out of denominator.

$$\frac{1}{x-1} - \frac{2}{x^2} = 0$$

$$(x-1)x^2\left(\frac{1}{x-1} - \frac{2}{x^2}\right) = 0 \cdot (x-1)x^2$$

dist law

$$\cancel{(x-1)}x^2 \cdot \frac{1}{\cancel{x-1}} - (x-1)\cancel{x^2} \cdot \frac{2}{\cancel{x^2}} = 0$$

frac law ① then ⑤

$$x^2 \boxed{-2}(x-1) = 0$$

dist law

$$x^2 - 2x + 2 = 0$$

$a = 1, \quad b = -2, \quad c = 2$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1}$$

$$= \frac{2 \pm \sqrt{4 - 8}}{2}$$

$$= \frac{2 \pm \sqrt{-4}}{2}$$

← if you got this far it counts as correct

$$= \frac{2 \pm i\sqrt{4}}{2}$$

$$= \frac{2 \pm 2i}{2}$$

GCF

$$= \frac{\cancel{2}(1 \pm i)}{\cancel{2}}$$

frac law ⑤

$$= \boxed{1 \pm i}$$